

Controlling information in probabilistic systems

The case of fault diagnosis

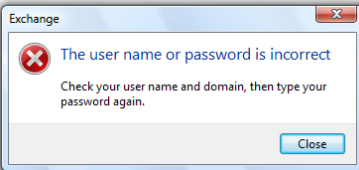
PhD defence of Engel Lefauchaux

Supervisors: Nathalie Bertrand and Serge Haddad

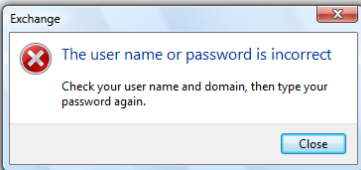
Septembre 24th 2018 – IRISA/LSV/Inria Rennes



Systems give information

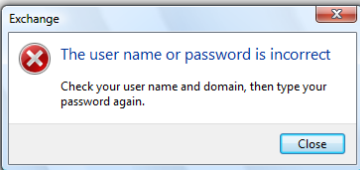


Systems give information



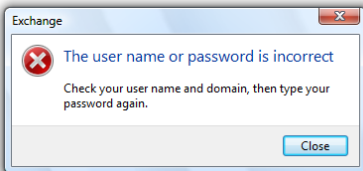
- Disclose useful information to the user

Systems give information



- Disclose useful information to the user
- Disclose secret information to an attacker

Systems give information



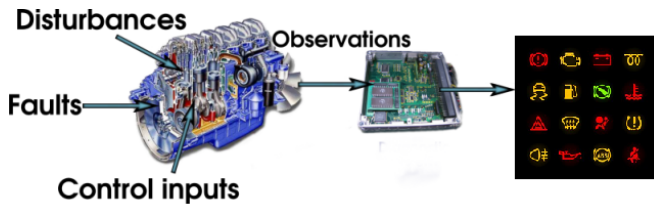
- Disclose useful information to the user
- Disclose secret information to an attacker

Analysing and controlling the revealed information is crucial

Detecting faulty behaviours

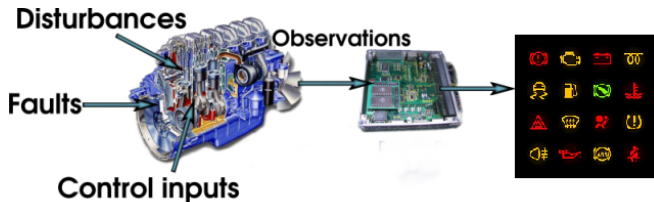
Fault diagnosis

Detecting faulty behaviours



Fault diagnosis

Detecting faulty behaviours



Diagnoser: must emit a verdict when faults occur, based on observations

Features of a diagnoser

Verdict: information provided

Correctness: accuracy of the verdict

Reactivity: delay before a verdict is given

Features of a diagnoser

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Diagnosability: does there exist a diagnoser?

Features of a diagnoser

Verdict: information provided

Correctness: accuracy of the verdict

Reactivity: delay before a verdict is given

Diagnosability: does there exist a diagnoser?

Synthesis: how to build a diagnoser?

Why diagnosis?

Faults and/or failures are unavoidable for some systems

- Components have a finite lifetime
- Reactive systems suffer from unpredictable behaviours of the environment



Why diagnosis?

Faults and/or failures are unavoidable for some systems

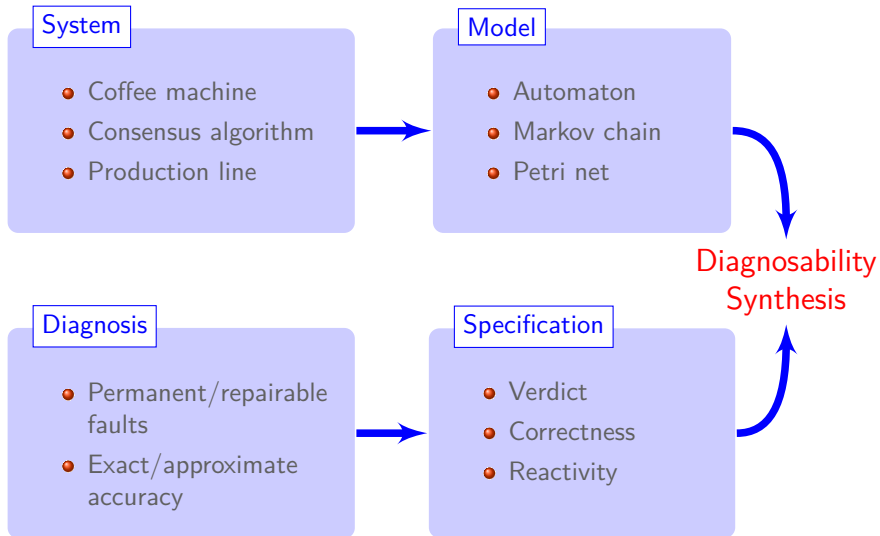
- Components have a finite lifetime
- Reactive systems suffer from unpredictable behaviours of the environment

Consequences of unhandled faults may be critical

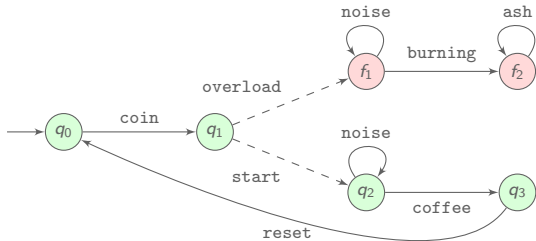
- Financial losses
- Human casualties



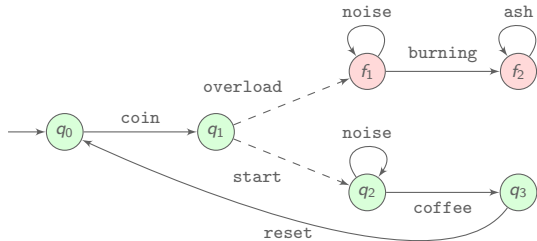
The model-based approach



A model for the coffee machine

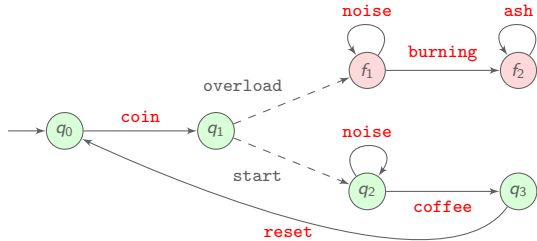


A model for the coffee machine



Partial observation

A model for the coffee machine

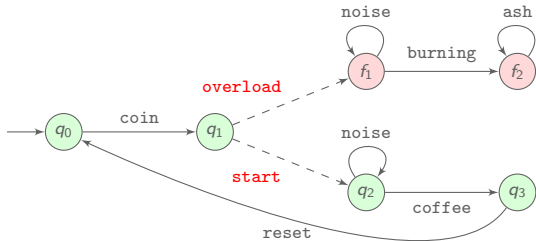


Partial observation

Observable events

→

A model for the coffee machine

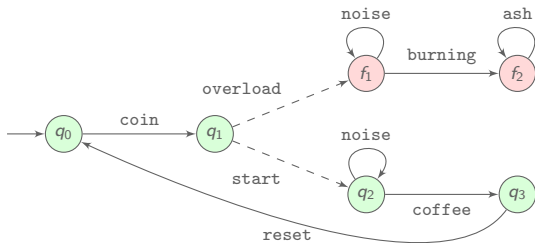


Partial observation

Observable events/unobservable events

\rightarrow / $--\rightarrow$

A model for the coffee machine



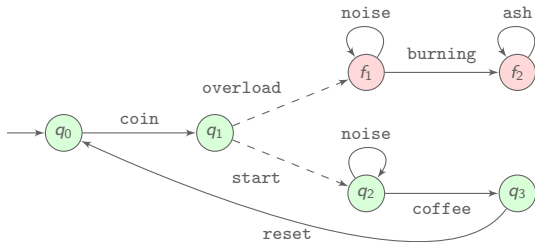
Partial observation

Observable events/unobservable events

\rightarrow / $--\rightarrow$

Run: $q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{\text{start}} q_2 \xrightarrow{\text{noise}} q_2$ | *Observation:* coin noise

A model for the coffee machine



Partial observation

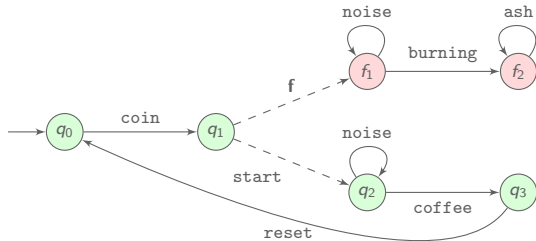
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One special event: A run is faulty iff the fault **f** occurs

A model for the coffee machine



Partial observation

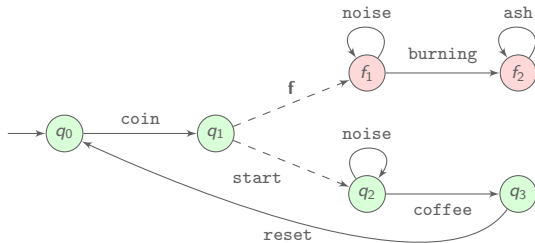
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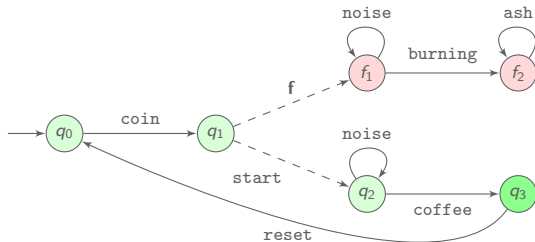
One special event: A run is faulty iff the fault **f** occurs

Classifying observations



Observation \longrightarrow Set of potential states of the system

Classifying observations

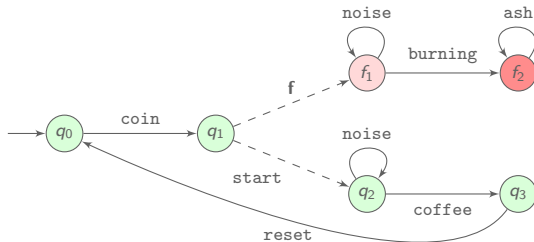


Observation \longrightarrow Set of potential states of the system

✓ coin coffee *surely correct*

$$\text{Obs}^{-1}(\text{coin coffee}) = \{q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{\text{start}} q_2 \xrightarrow{\text{coffee}} q_3\}$$

Classifying observations

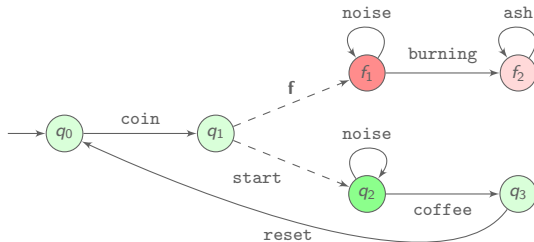


Observation \longrightarrow Set of potential states of the system

- ✓ coin coffee *surely correct*
- ✗ coin burning *surely faulty*

$$\text{Obs}^{-1}(\text{coin burning}) = \{q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{f} f_1 \xrightarrow{\text{burning}} f_2\}$$

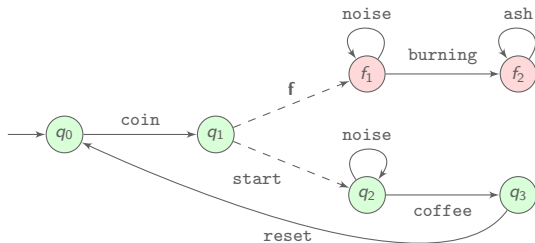
Classifying observations



Observation \longrightarrow Set of potential states of the system

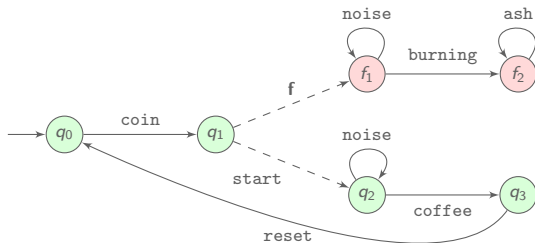
- ✓ coin coffee *surely correct*
- ✗ coin burning *surely faulty*
- ? coin noise *ambiguous*

$$\text{Obs}^{-1}(\text{coin noise}) = \{ q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{\text{start}} q_2 \xrightarrow{\text{noise}} q_2, q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{f} f_1 \xrightarrow{\text{noise}} f_1 \}$$



- **Verdict:** detection of faults
- **Correctness:** if a fault is claimed, a fault occurred
- **Reactivity:** every fault will be detected after a bounded delay

[SSLST96] Sampath, Sengupta, Lafortune, Sinnamohideen and Teneketzis, *Failure diagnosis using discrete-event models*, IEEE TCST, 1996.

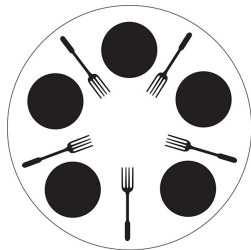


- **Verdict:** detection of faults
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Correct but not reactive diagnoser: claiming a fault when burning occurs

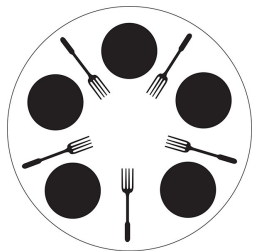
[SSLST96] Sampath, Sengupta, Lafortune, Sinnamohideen and Teneketzis, *Failure diagnosis using discrete-event models*, IEEE TCST, 1996.

Useful to model some systems

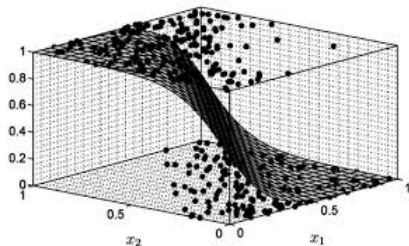


Internal random behaviour

Useful to model some systems

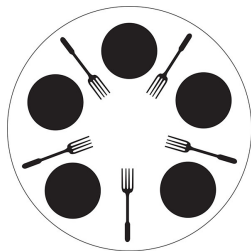


Internal random behaviour

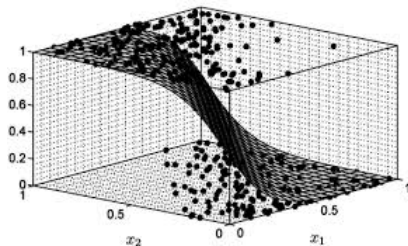


Models through statistical analysis

Useful to model some systems



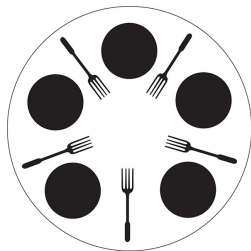
Internal random behaviour



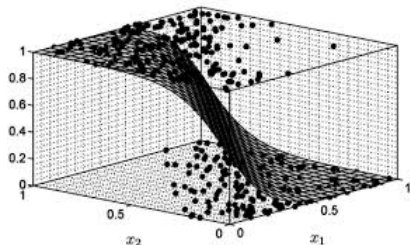
Models through statistical analysis

Enable quantitative requirements

Useful to model some systems



Internal random behaviour

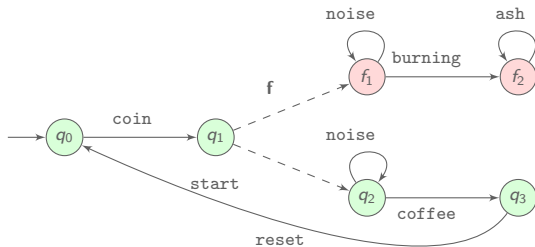


Models through statistical analysis

Enable quantitative requirements

- Is the system diagnosable, ignoring negligible behaviours?
- What is the measure of undetected faults?
- What is the average delay of fault detection?

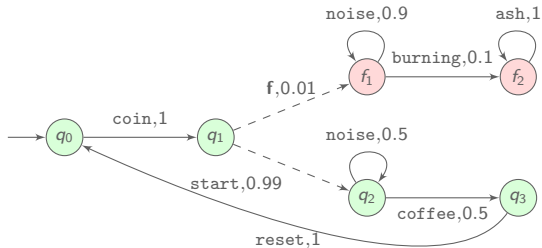
A probabilistic model for the coffee machine



Probability of a run

$$\text{Run } \rho = q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{\text{start}} q_2 \xrightarrow{\text{noise}} q_2$$

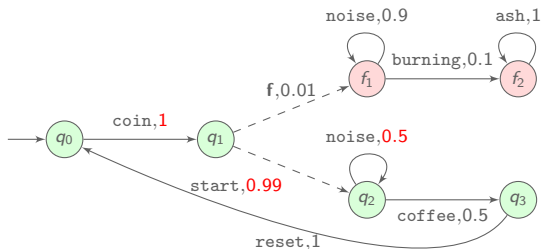
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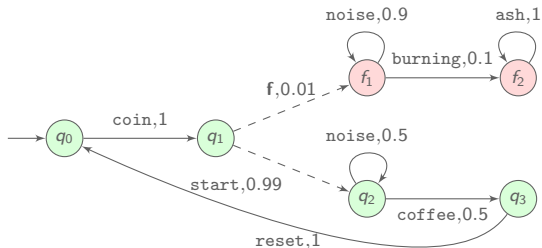
A probabilistic model for the coffee machine



Probability of a run

Run $\rho = q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{\text{start}} q_2 \xrightarrow{\text{noise}} q_2$ | Probability $\mathbb{P}(\rho) = 1 \times 0.99 \times 0.5$

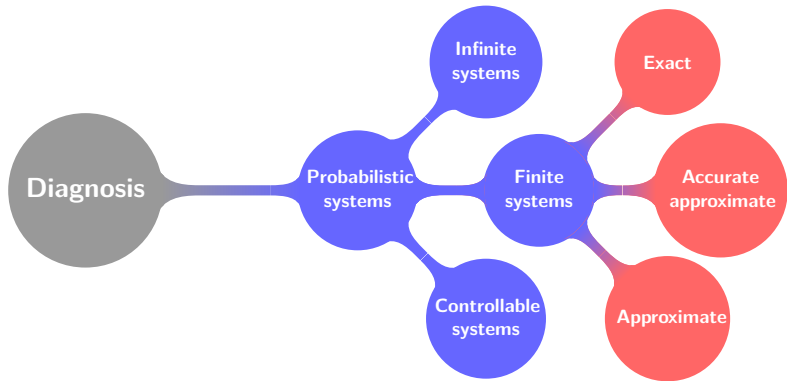
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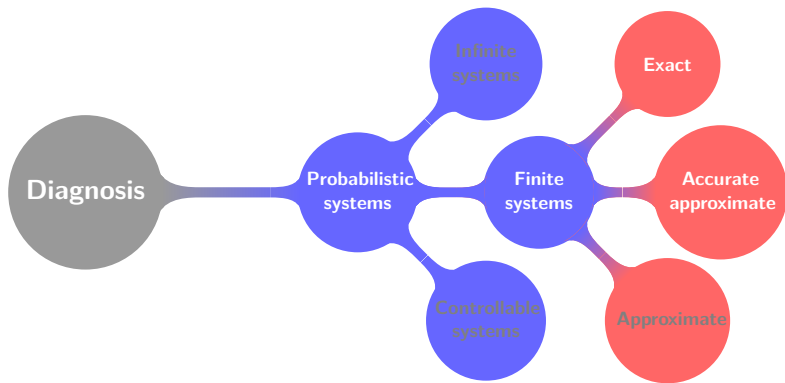


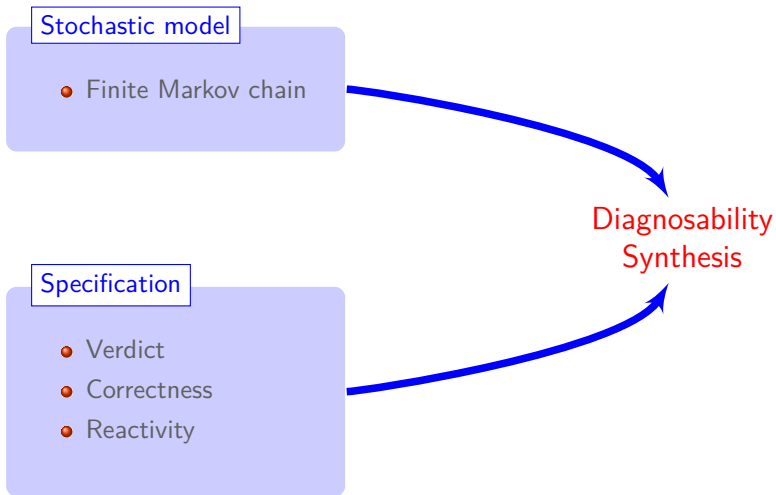
Probability of a run

Run $\rho = q_0 \xrightarrow{\text{coin}} q_1 \xrightarrow{\text{start}} q_2 \xrightarrow{\text{noise}} q_2$ | Probability $\mathbb{P}(\rho) = 1 \times 0.99 \times 0.5$

→ Defines a probability measure on the set of infinite runs







Stochastic model

- Finite Markov chain

Specification

- Verdict

Goal 1: Formalise and compare the specifications for stochastic systems

Diagnosability
Synthesis

Challenges

Stochastic model

- Finite Markov chain

Goal 2: Decidability and complexity of diagnosability and synthesis

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Specification

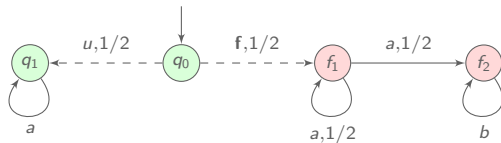
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Goal 1: Formalise and compare the specifications for stochastic systems

Diagnosability
Synthesis

Many possible specifications

Verdict: information provided

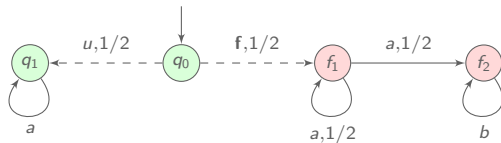


Correctness: accuracy of the verdict

Reactivity: delay before a verdict is given

Many possible specifications

Verdict: information provided



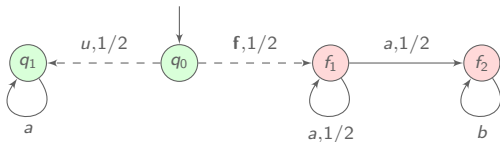
a^n is ambiguous

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a^n is ambiguous

Faults are detected almost surely:

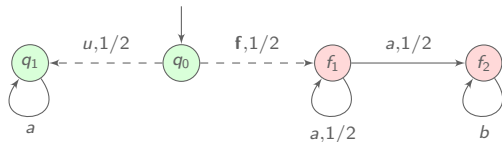
$$\lim_{n \rightarrow \infty} \mathbb{P}(\{q_0 \xrightarrow{f} f_1 (\xrightarrow{a} f_1)^n, q_0 \xrightarrow{f} (f_1 \xrightarrow{a})^n f_2\}) = 0$$

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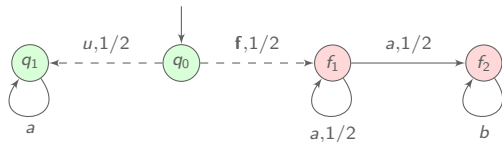
Correct runs stay ambiguous: $\lim_{n \rightarrow \infty} \mathbb{P}(\{q_0 \xrightarrow{u} q_1 (\xrightarrow{a} q_1)^n\}) = \frac{1}{2}$

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Correct runs stay ambiguous: $\lim_{n \rightarrow \infty} \mathbb{P}(\{q_0 \xrightarrow{u} q_1 (\xrightarrow{a} q_1)^n\}) = \frac{1}{2}$

Faulty runs or all ambiguous runs?

Correctness: accuracy of the verdict

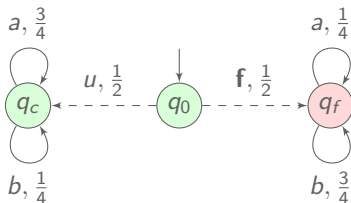
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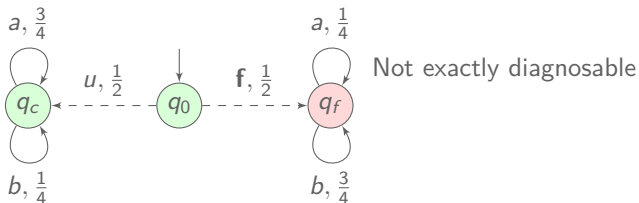
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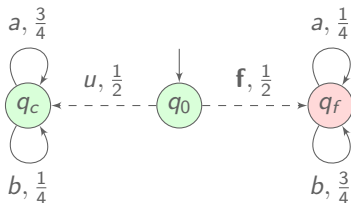
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Not exactly diagnosable

However a high proportion of b implies a highly probable faulty run

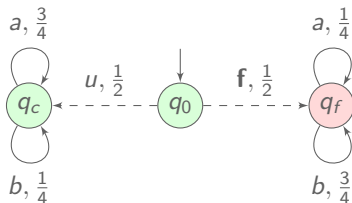
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Exact, approximate or accurate approximate?

Reactivity: delay before a verdict is given

Many possible specifications

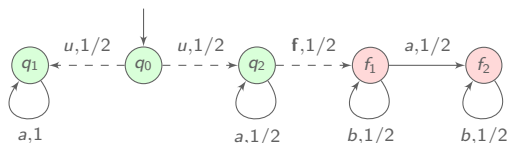
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Many possible specifications

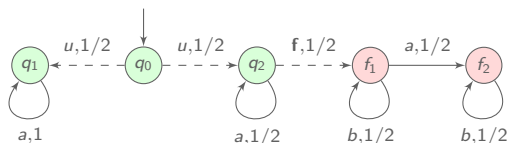
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a^n is likely to be observed $\mathbb{P}(a^n) = \frac{1}{2} + \frac{1}{2^n} + \frac{1}{2^{n-1}}$

Many possible specifications

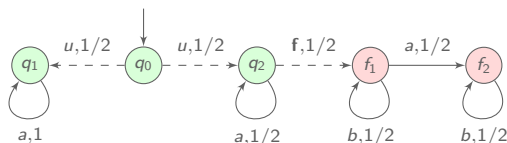
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a^n is likely to be observed $\mathbb{P}(a^n) = \frac{1}{2} + \frac{1}{2^n} + \frac{1}{2^{n-1}}$

However, a^ω is surely correct: $q_0 \xrightarrow{u} q_1 (\xrightarrow{a} q_1)^\omega$, $q_0 \xrightarrow{u} q_2 (\xrightarrow{a} q_2)^\omega$

Many possible specifications

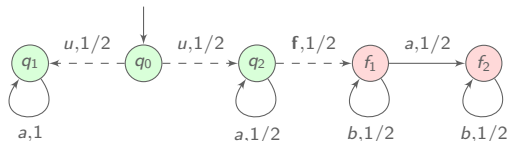
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Almost sure, uniform almost sure or infinite?

Many possible specifications

Verdict: information provided

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Exact, approximate or accurate approximate?

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Almost sure, uniform almost sure or infinite?

Each combination of features defines a diagnoser notion

Many possible specifications

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Almost sure, uniform almost sure or infinite?

Each combination of features defines a diagnoser notion

→ Semantical analysis of the relations

Formalising diagnosability notions

Verdict: Detection of faulty runs

Reactivity: Finite delay

Formalising diagnosability notions

Verdict: Detection of faulty runs

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Reactivity Correctness	Uniform almost sure	Almost sure
Exact	Uniform FF-diagnosability	FF-diagnosability $\lim_{n \rightarrow \infty} \mathbb{P}(FAmb_n) = 0$
Accurate approximate	Uniform AFF-diagnosability	AFF-diagnosability $\forall \varepsilon > 0, \lim_{n \rightarrow \infty} \mathbb{P}(FAmb_n^\varepsilon) = 0$
Approximate	Uniform ε FF-diagnosability	ε FF-diagnosability

Formalising diagnosability notions

Verdict: Detection of faulty runs

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Reactivity Correctness	Uniform almost sure	Almost sure
Exact	Uniform FF-diagnosability [TT05][BHL18]	FF-diagnosability [BHL14][BHL18] $\lim_{n \rightarrow \infty} \mathbb{P}(FAmb_n) = 0$
Accurate approximate	Uniform AFF-diagnosability [TT05][BHL16][BHL18]	AFF-diagnosability [BHL16][BHL18] $\forall \varepsilon > 0, \lim_{n \rightarrow \infty} \mathbb{P}(FAmb_n^\varepsilon) = 0$
Approximate	Uniform ε FF-diagnosability [BHL16][BHL18]	ε FF-diagnosability [BHL16][BHL18]

[TT05] Thorsley and Teneketzis, *Diagnosability of stochastic discrete-event systems*, IEEE TAC, 2005.

[BHL14] Bertrand, Haddad and Lefaucheu, *Foundation of diagnosis and predictability in probabilistic systems*, FSTTCS, 2014.

[BHL16] Bertrand, Haddad and Lefaucheu, *Accurate approximate diagnosability of stochastic systems*, LATA, 2016.

[BHL18] Bertrand, Haddad and Lefaucheu, *A Tale of Two Diagnoses in Probabilistic Systems*, I&C, 2018.

Formalising diagnosability notions

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	Reactivity	Uniform almost sure	Almost sure
Correctness			
Exact		Uniform FF-diagnosability [TT05][BHL18]	FF-diagnosability [BHL14][BHL18] $\lim_{n \rightarrow \infty} \mathbb{P}(FAmb_n) = 0$
		⇓	⇓
Accurate approximate		Uniform AFF-diagnosability [TT05][BHL16][BHL18]	AFF-diagnosability [BHL16][BHL18] $\forall \varepsilon > 0, \lim_{n \rightarrow \infty} \mathbb{P}(FAmb_n^\varepsilon) = 0$
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Approximate		Uniform ε FF-diagnosability [BHL16][BHL18]	ε FF-diagnosability [BHL16][BHL18]

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Formalising diagnosability notions

Verdict: Detection of faulty runs

Reactivity: Finite delay

	Reactivity	Uniform almost sure	Almost sure
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Exact		Uniform FF-diagnosability [TT05][BHL18]	FF-diagnosability [BHL14][BHL18] $\lim_{n \rightarrow \infty} \mathbb{P}(FAmb_n) = 0$
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Challenges

Stochastic model

- Finite Markov chain

Goal 2: Decidability and complexity of diagnosability and synthesis

Specification

- Verdict

Goal 1: Formalise and compare the specifications for stochastic systems

Diagnosability
Synthesis

Challenges

Stochastic model

- Finite Markov chain

Goal 2: Decidability and complexity of diagnosability and synthesis

Specification

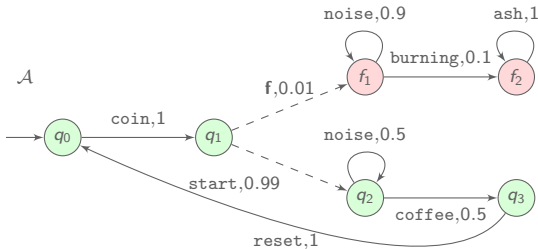
- Verdict

Goal 1: Formalise and compare the specifications for stochastic systems

Diagnosability
Synthesis

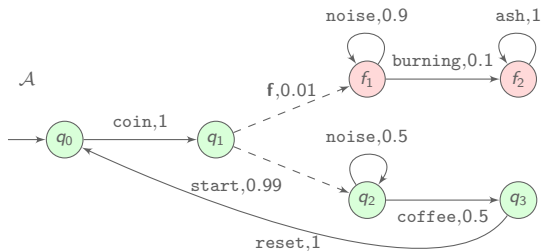
Definition

The probability of faulty ambiguous runs converges to 0



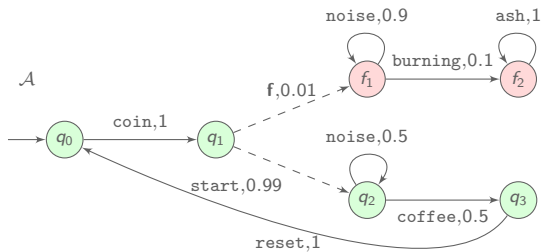
Faulty ambiguous runs: ending with $(f_1 \xrightarrow{\text{noise}} f_1)^*$

The belief construction



$\mathcal{O}_{\mathcal{A}}$: sequence of observations \mapsto set of possible current states

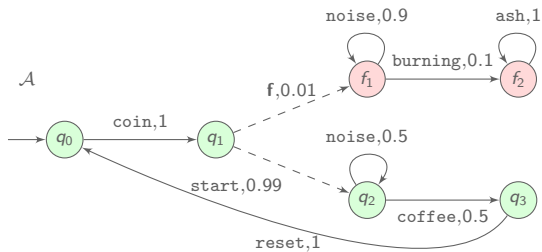
The belief construction



$\mathcal{O}_{\mathcal{A}}$



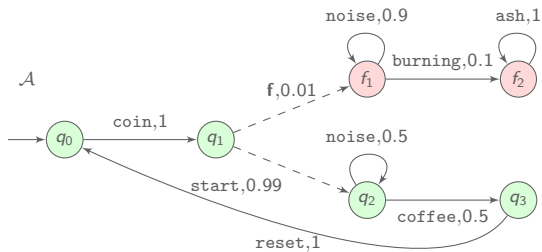
The belief construction



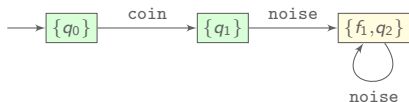
\mathcal{O}_A



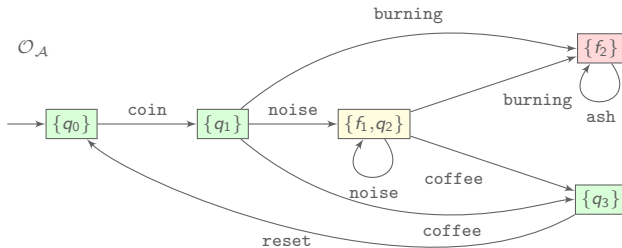
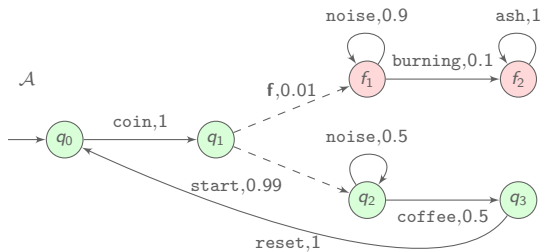
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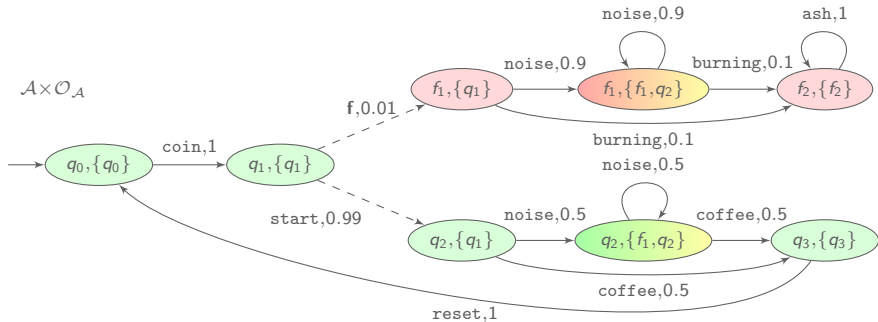
\mathcal{O}_A



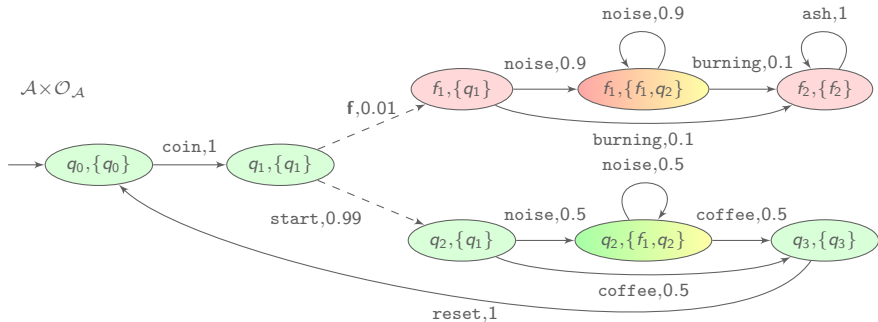
The belief construction



Synchronised product

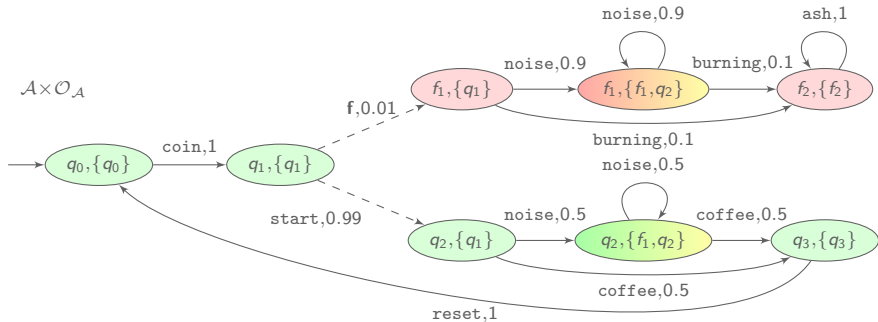


Synchronised product



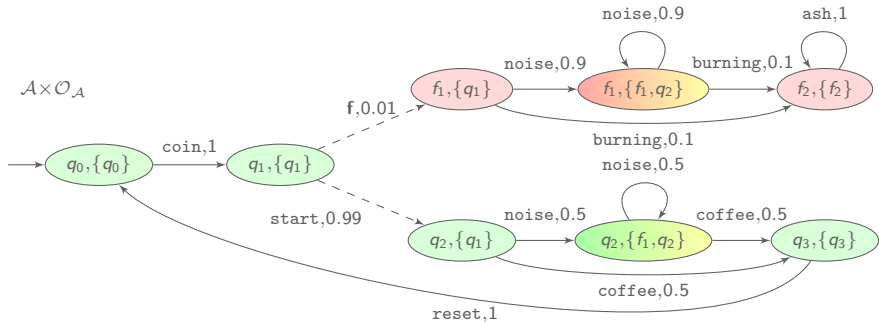
- Same stochastic behaviour as \mathcal{A}

Synchronised product



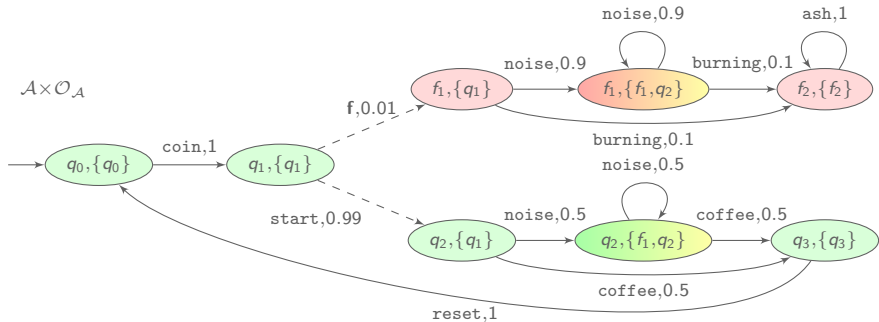
- Same stochastic behaviour as \mathcal{A}
- Ambiguity of a run deduced from its last state

Synchronised product



- Same stochastic behaviour as \mathcal{A}
- Ambiguity of a run deduced from its last state
- Possibly exponential in the size of \mathcal{A}

Synchronised product



- Same stochastic behaviour as \mathcal{A}
- Ambiguity of a run deduced from its last state
- Possibly exponential in the size of \mathcal{A}

FF-diagnosable iff
no BSCC contains a faulty ambiguous state

Complexity of FF-diagnosability

Verdict: Detection of faulty runs

Reactivity: Finite delay

Reactivity Correctness	Uniform almost sure	Almost sure
Exact	Uniform FF-diagnosability [TT05] [BHL18]	FF-diagnosability PSPACE-complete [BHL14][BHL18]
Accurate approximate	Uniform AFF-diagnosability [TT05][BHL16][BHL18]	AFF-diagnosability [BHL16][BHL18]
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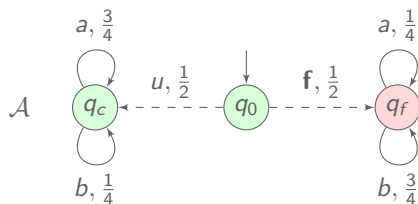
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Definition

Faults are almost surely detected with arbitrarily small probability of false positive

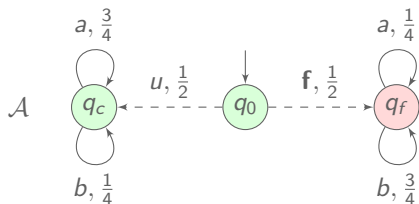


$$\mathbb{P}(\text{correct} \mid bba) = \frac{1}{4}$$

$$\mathbb{P}(\text{correct} \mid bbab) = \frac{1}{10}$$

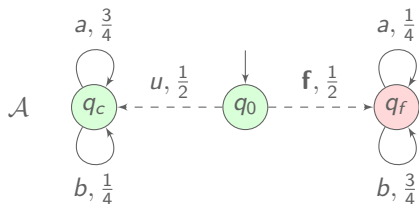
High proportion of $b \Rightarrow$ small probability of being correct

Distance between states



Distance between q_c and q_f : $\sup_{E \subseteq \{a,b\}^\omega} \mathbb{P}_{q_f}(E) - \mathbb{P}_{q_c}(E)$.

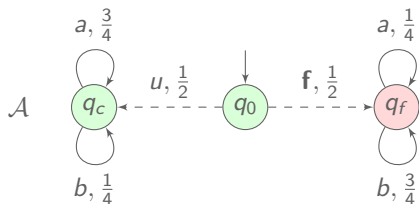
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$E = \{\text{infinite words with proportion of } b \text{ greater than half}\}$

Distance between states

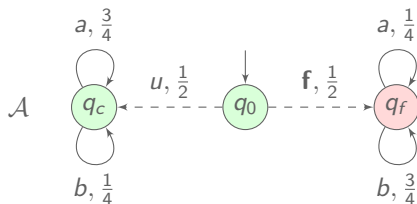


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E separates q_c and q_f : $\mathbb{P}_{q_c}(E) = 0$ and $\mathbb{P}_{q_f}(E) = 1$

Distance between states



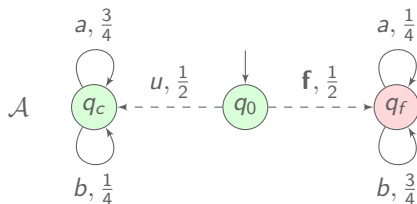
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→ The distance between q_c and q_f is 1

Distance between states



Distance between q_c and q_f : $\sup_{E \subseteq \{a,b\}^\omega} \mathbb{P}_{q_f}(E) - \mathbb{P}_{q_c}(E)$.

$E = \{\text{infinite words with proportion of } b \text{ greater than half}\}$

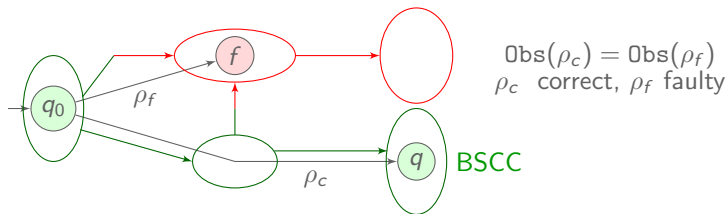
E separates q_c and q_f : $\mathbb{P}_{q_c}(E) = 0$ and $\mathbb{P}_{q_f}(E) = 1$

→ The distance between q_c and q_f is 1

→ \mathcal{A} is AFF-diagnosable

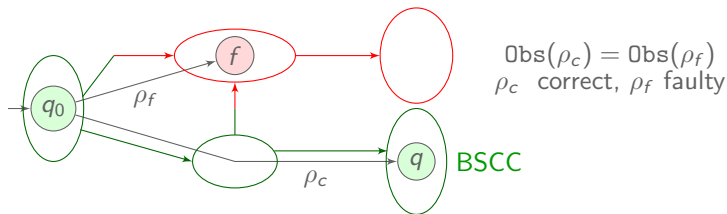
Solving AFF-diagnosability

- Identifying **relevant pairs** of states



Solving AFF-diagnosability

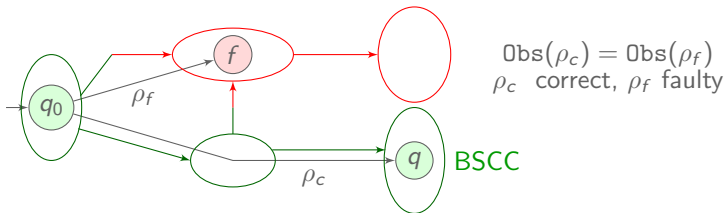
- Identifying **relevant pairs** of states



- Checking distance 1 for all relevant pairs

Solving AFF-diagnosability

- Identifying **relevant pairs** of states

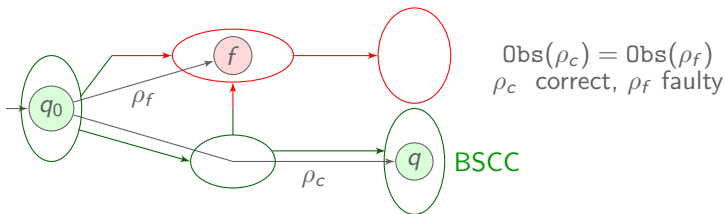


- Checking distance 1 for all relevant pairs

AFF-diagnosable iff distance 1 for all relevant pairs

Solving AFF-diagnosability

- Identifying **relevant pairs** of states



- Checking distance 1 for all relevant pairs

AFF-diagnosable iff distance 1 for all relevant pairs

- The distance 1 problem is in PTIME [CK14]

[CK14] Chen and Kiefer, *On the Total Variation Distance of Labelled Markov Chains*, CSL-LICS, 2014.

Complexity of AFF-diagnosability

Verdict: Detection of faulty runs

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Reactivity Correctness	Uniform almost sure	Almost sure
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Challenges

Stochastic model

- Finite Markov chain

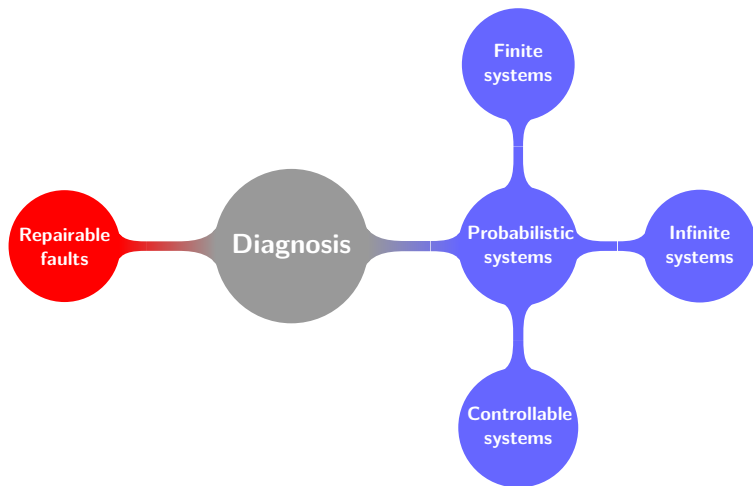
Goal 2: Decidability and complexity of diagnosability and synthesis

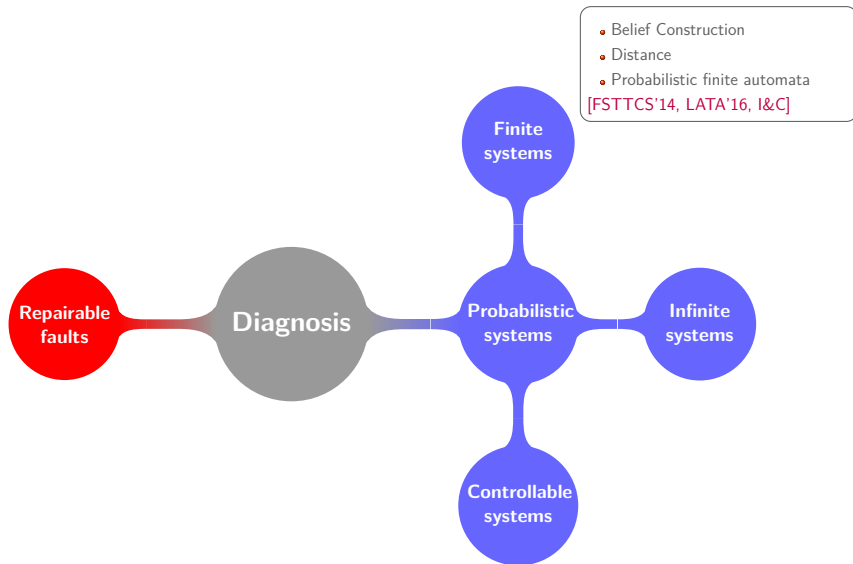
Specification

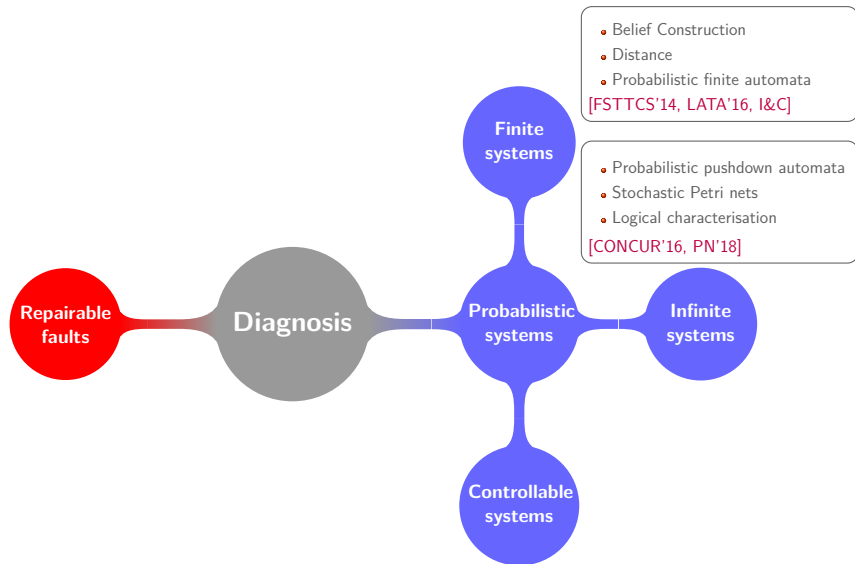
- Verdict

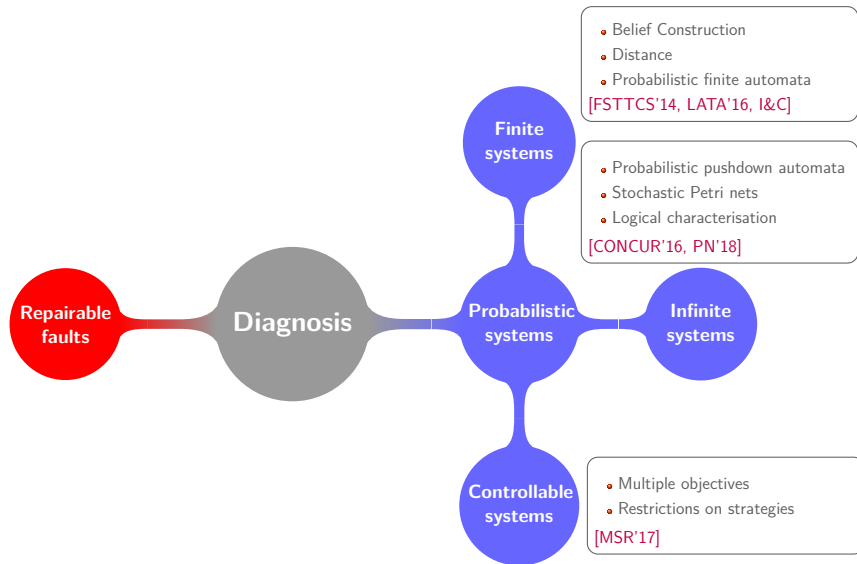
Goal 1: Formalise and compare the specifications for stochastic systems

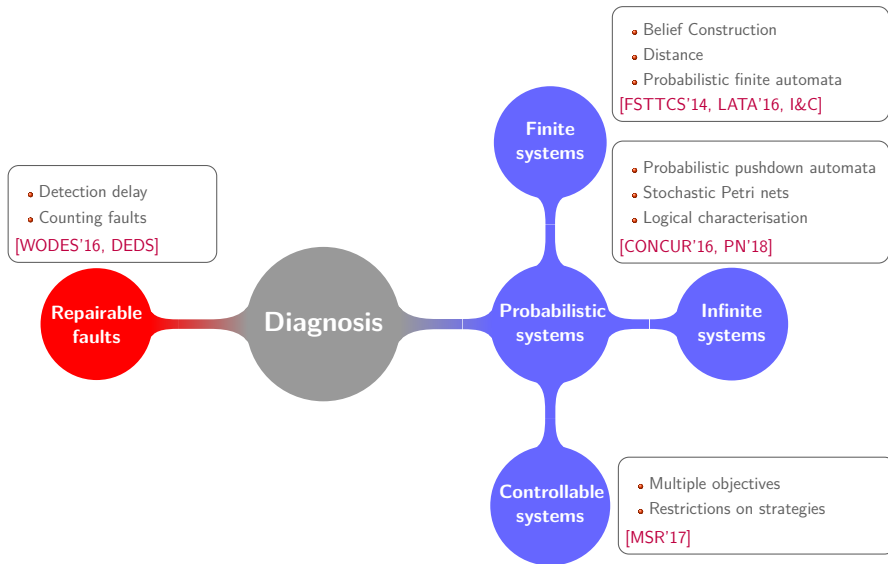
Diagnosability
Synthesis

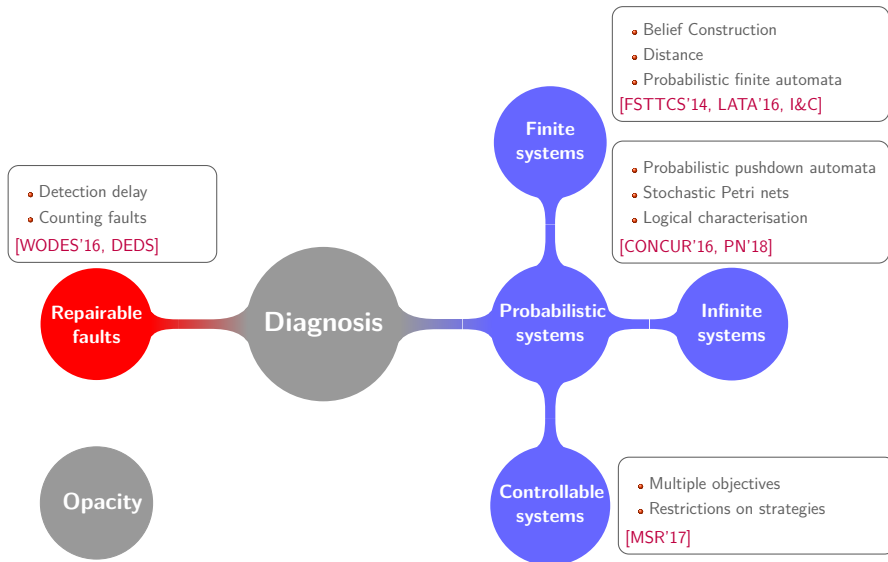


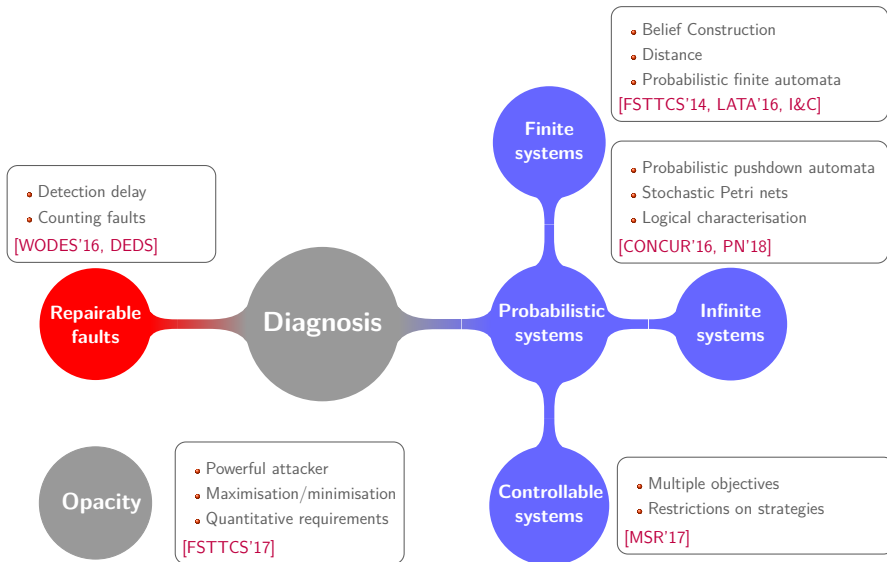


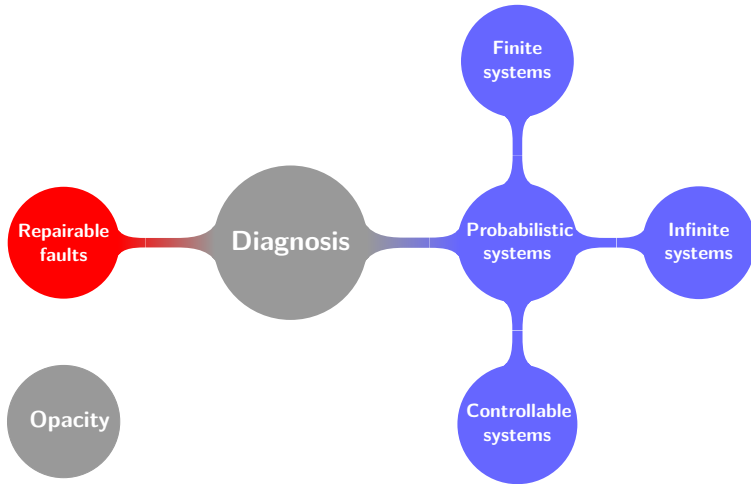


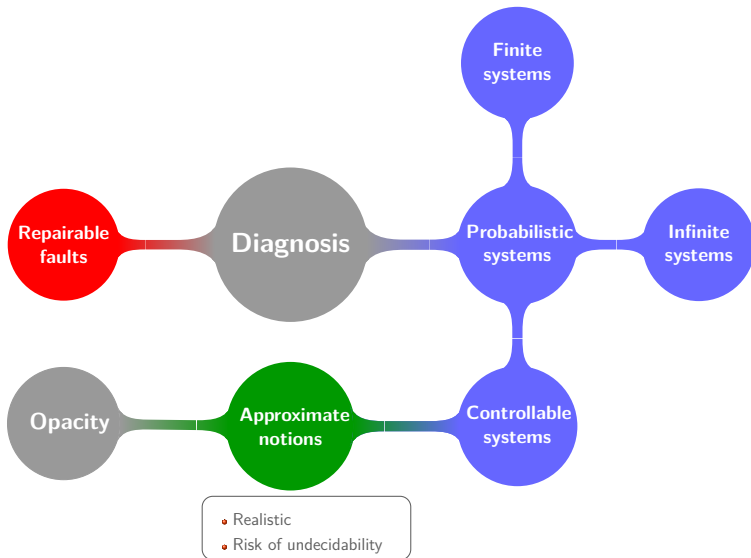




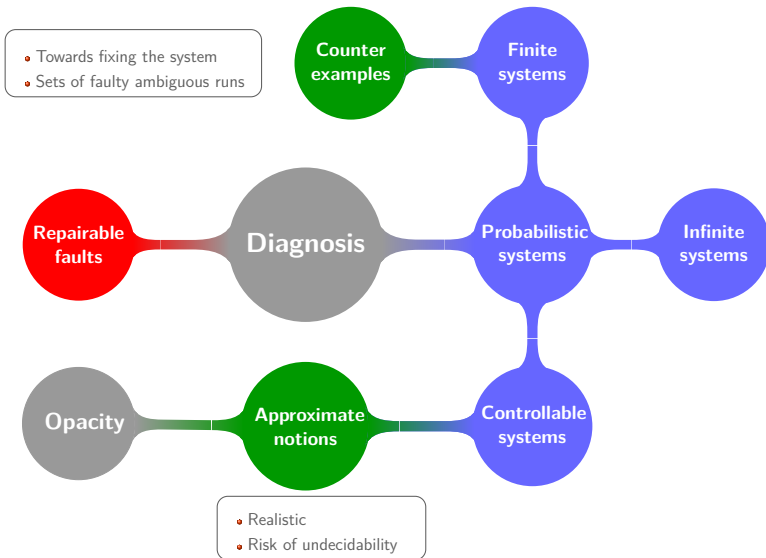








Perspectives



Perspectives

